Generation of generalized 3-regular graphs

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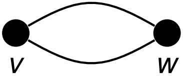
Which structures will be generated?

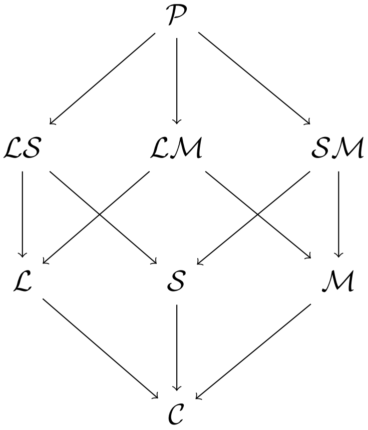
3-regular variety of

- simple graphs
- multigraphs
- graphs with loops
- graphs with semi-edges
- any combination of these

Which structures will be generated?

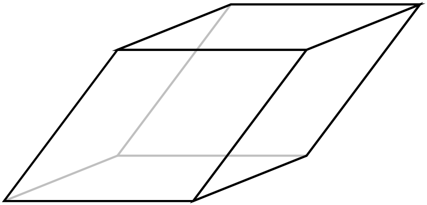
Name	Type	Counts as
Loop		2
Multi-edge		2
Semi-edge		1

Which structures will be generated?



Motivation

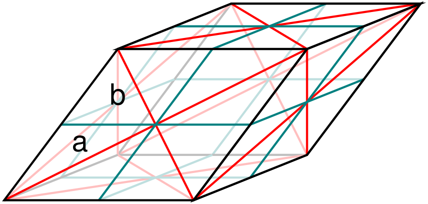
- Study of maps
  - flag graphs of maps / hypermaps
  - symmetry type graphs / Delaney-Dress graphs
  - arc graphs of oriented maps
- Voltage graphs

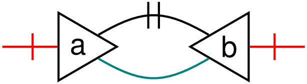


Rhombohedron

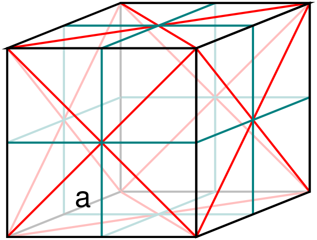
all 6 faces are congruent rhombi
has D_{3d} symmetry
(Trigonal trapezohedron)

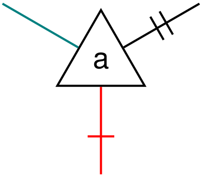
Motivation - Delaney-Dress graph





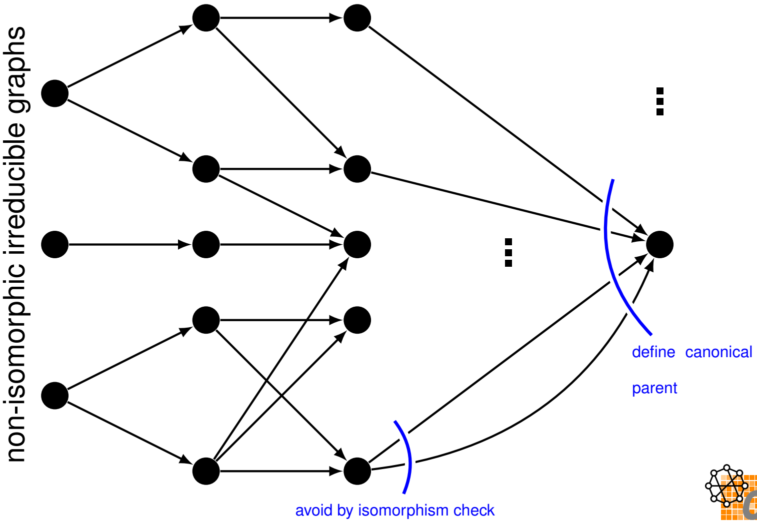
Motivation - Delaney-Dress graph





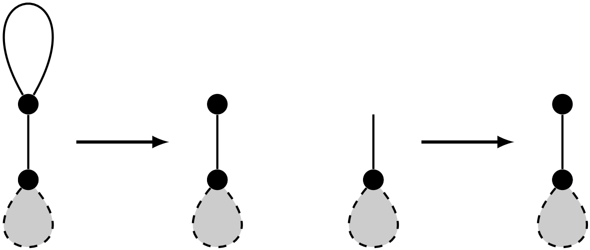
Which technique will be used?

Canonical construction path

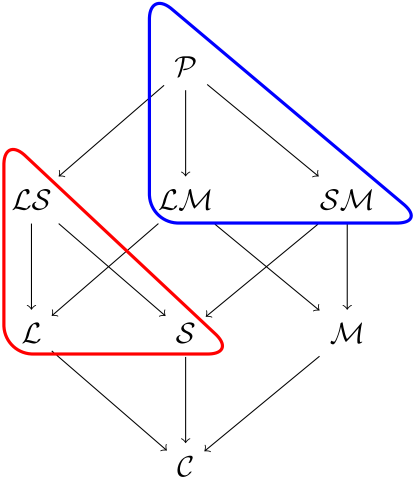


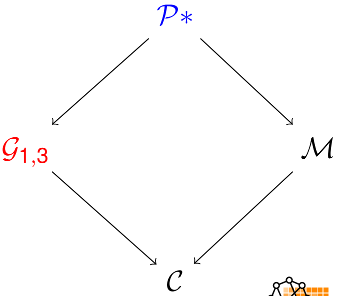
Pregraph primitives

Translate cubic pregraphs to multigraphs with degrees 1 and 3.
Notation: $*(G)$ is the primitive of G.

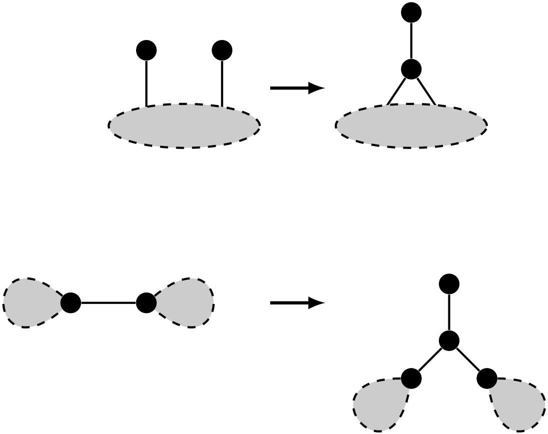


Translation to multigraphs

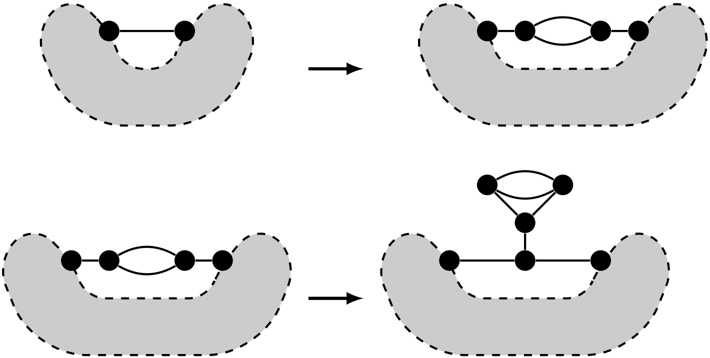




Which are the construction operations?

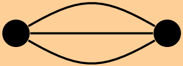


Which are the construction operations?

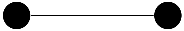
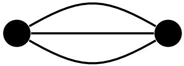
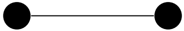
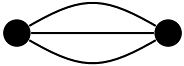


What are the irreducible graphs?

Use inverse operation to reduce each pregraph primitive

$\Rightarrow$ each pregraph primitive can be reduced to a cubic simple graph,  or 

What are the irreducible graphs?

Target class	Irreducible graphs
$\mathcal{C}$	$\mathcal{C}$
$\mathcal{G}_{1,3}$	$\mathcal{C}$ 
$\mathcal{M}$	$\mathcal{C}$ 
$\mathcal{P}*$	$\mathcal{C}$  

What are the irreducible graphs?

- degree 1 vertices don't count towards the order of the graph when translating from $\mathcal{G}_{1,3}$ to $\mathcal{S}$ (and similar)
- number of degree 3 vertices never decreases when applying the construction operations

- $\mathcal{L}$,$\mathcal{M}$,$\mathcal{LM}$ with n vertices $\rightarrow \mathcal{C}$ with $\leq n$ vertices.
- $\mathcal{S}$,$\mathcal{LS}$,$\mathcal{SM}$,$\mathcal{LSM}$ with n vertices $\rightarrow \mathcal{C}$ with $\leq n$ vertices, but intermediate $\mathcal{G}_{1,3}$ and $\mathcal{P}*$ with $\leq 2n + 2$ vertices

Translation from $\mathcal{G}_{1,3}$ to $\mathcal{L}$, $\mathcal{S}$ and $\mathcal{LS}$

$\mathcal{G}_{1,3}(n)$ to $\mathcal{L}$(n)

translation always possible

$\mathcal{G}_{1,3}(\leq 2n+2)$ to $\mathcal{S}(n)$

$\forall G \in \mathcal{G}_{1,3}(\leq 2n+2) : V_3(G) = n \Rightarrow \exists! G' \in \mathcal{S}(n) : *(G') = G$

$\mathcal{G}_{1,3}(\leq 2n+2)$ to $\mathcal{LS}(n)$

$\forall G \in \mathcal{G}_{1,3}(\leq 2n+2) : V(G) \geq n \wedge V_3(G) \leq n \Rightarrow \exists G' \in \mathcal{LS}(n) : *(G') = G$

$n - V_3(G)$ vertices of degree 1 correspond to vertices with loops, rest corresponds to semi-edges (homomorphism principle)

Results and timings

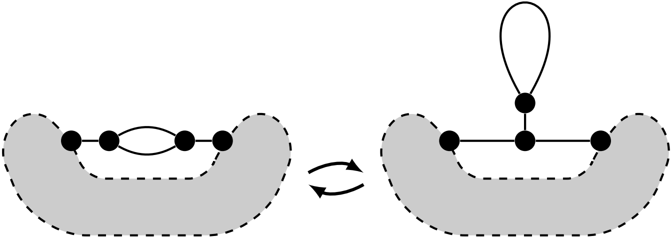
	C	L	S	M	LS	LM	SM	LSM
1	0	0	1	0	2	0	1	2
2	0	1	1	1	3	2	3	5
3	0	0	2	0	4	0	4	7
4	1	2	6	2	12	5	12	22
5	0	0	10	0	22	0	22	43
6	2	6	29	6	68	17	68	141
7	0	0	64	0	166	0	166	373
8	5	20	194	20	534	71	534	1270
9	0	0	531	0	1589	0	1589	4053
10	19	91	1733	91	5464	388	5464	14671
11	0	0	5524	0	18579	0	18579	52826
12	85	509	19430	509	68320	2592	68320	203289
13	0	0	69322	0	255424	0	255424	795581
14	509	3608	262044	3608	1000852	21096	1000852	3241367
15	0	0	1016740	0	4018156	0	4018156	13504130
16	4060	31856	4101318	31856	16671976	204638	16671976	57904671
17	0	0	16996157	0	70890940	0	70890940	253856990
18	41301	340416	72556640	340416	309439942	2317172	309439942	1139231977

Results and timings

	L	S	M	LS	LM	SM	LSM
1	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s
2	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s
3	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s
4	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s
5	0m0.000s	0m0.000s	0m0.000s	0m0.004s	0m0.000s	0m0.000s	0m0.000s
6	0m0.004s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s
7	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.000s	0m0.004s	0m0.004s
8	0m0.000s	0m0.004s	0m0.000s	0m0.004s	0m0.004s	0m0.012s	0m0.012s
9	0m0.000s	0m0.016s	0m0.000s	0m0.020s	0m0.000s	0m0.028s	0m0.036s
10	0m0.000s	0m0.048s	0m0.004s	0m0.068s	0m0.004s	0m0.104s	0m0.128s
11	0m0.000s	0m0.164s	0m0.000s	0m0.244s	0m0.000s	0m0.384s	0m0.492s
12	0m0.008s	0m0.648s	0m0.012s	0m0.956s	0m0.032s	0m1.516s	0m2.028s
13	0m0.000s	0m2.408s	0m0.000s	0m3.860s	0m0.000s	0m6.096s	0m8.429s
14	0m0.052s	0m9.669s	0m0.072s	0m16.509s	0m0.288s	0m25.606s	0m36.978s
15	0m0.000s	0m39.906s	0m0.000s	1m12.645s	0m0.000s	1m49.883s	2m45.270s
16	0m0.520s	2m50.527s	0m0.724s	5m26.200s	0m3.104s	8m10.459s	12m47.968s
17	0m0.000s	12m26.539s	0m0.000s	25m34.240s	0m0.000s	36m54.106s	61m11.377s
18	0m6.068s	56m0.610s	0m8.577s	123m19.194s	0m39.026s	169m9.242s	300m11.002s

2.40 GHz Intel Xeon

Connection loops and multi-edges



Future work

- 3-edge-colorable graphs
- 3-edge-colored graphs
- graphs with a 2-factor where each component is a quotient of a 4-cycle
- graphs with a 2-factor where each component is a 4-cycle

Thank you for your attention